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STRESS-STRAIN BEHAVIOR AND FAILURE OF UNIAXIAL COMPOSITES IN

COMBINED COMPRESSION AND SHEAR - II

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This paper presents a theoretical approach to the problem of finding the stress-strain relations for composites under combined compression and shear. To this end, the 1-element model, presented in Part I of this paper, is replaced by a 2-element model. Both elastic and inelastic composites are explored employing graphical methods. A numerical example is worked out to illustrate the effects of combined loads on the stiffness and strength of a composite.

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20. ABSTRACT

The present theory employs a nonlinear algebraic equation. An alternative approach would be to use a linear incremental theory, leading to a linear differential equation. An analysis is offered to show that the latter approach leads to substantially the same results as the 1-element model.

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STRESS-STRAIN BEHAVIOR AND FAILURE OF UNIAXIAL COMPOSITES IN COMBINED COMPRESSION AND SHEAR - II

INTRODUCTION

Part I of this paper [1] presented a unified approach to the title problem. To account for the fact that uniaxially reinforced composites fail in axial compression at stresses far below the theoretical microbuckling stress it assumed (as previous authors have done) that failure is due to the combined effects of initial fiber misalignment, or tilt, and nonlinear shear response of the matrix material. The behavior of a single misaligned volume element of the composite was worked out, and was used to find the compressive stress-strain relation and the maximum compressive stress that a composite can support.

A 1-element model is unsuitable for finding the shear strain in a composite under load, since pure axial compression, for instance, would increase the tilt and thus predict a spurious shear strain. This difficulty can be avoided by employing a 2-element model, in which two identical elements have equal and opposite tilt. By this means we eliminate the spurious shear strain in pure compression while retaining the merit of the initial tilt in accounting for the low compressive strength and nonlinear strain response of the composite.

One might ask, why go to the trouble of developing such a theory? Why not transform the stresses calculated for the load system of coordinates into stresses in the material principal axis system, use the (assumed known) material properties to find the strains in the material coordinate system, and then transform these strains back to the load system of coordinates? This is an established approach in linear elastic theory for anisotropic materials; why not use it here?

One reason is that because the material response is nonlinear it would be necessary to know the strains for all combinations of shear and compressive stress in the material coordinate system. Presently such information is unavailable. A second reason is that the theory discussed herein provides a much less cumbersome method of achieving the desired goal.

Part II, then, deals with the properties of a 2-element model. We first deal with the elastic case, and show how the shear strains can be obtained graphically. We then discuss the inelastic case and show how to find the stress-strain relations and strength interaction between compression and shear for proportional loading.

1

ELASTIC COMPOSITE

Let us assume that two otherwise identical kink band elements a and b have initial inclinations $-\alpha_0$ and α_0 respectively (see Fig.1). We further assume that they are subjected to the same shear stress τ and compressive stress σ, where $\tau = k\sigma$ (proportional loading case). Then using the theory of Part I

$$\gamma_b = \frac{(k + \alpha_0)\sigma}{G - \sigma} \tag{1}$$

$$\gamma_a = \frac{(k - \alpha_0)\sigma}{G - \sigma} \tag{2}$$

where γ is the change in tilt angle caused by the action of the applied stresses. We assume additionally that the strains in the composite as a whole are the averages of the strains obtained for the two elements. Then

$$\gamma = \bar{\gamma} = \frac{k\sigma}{G - \sigma} \tag{3}$$

$$\varepsilon = \bar{\varepsilon} = \frac{\sigma}{E} + \frac{1}{2}(\varepsilon_a + \varepsilon_b) \tag{4a}$$

$$\approx \frac{\sigma}{E} + \frac{1}{2}\left[-\alpha_0\gamma_a + \frac{1}{2}\gamma_a^2 + \alpha_0\gamma_b + \frac{1}{2}\gamma_b^2 \right] \tag{4b}$$

$$= \frac{\sigma}{E} + \frac{\alpha_0^2\sigma}{G - \sigma} + \frac{(\alpha_0^2 + k^2)\sigma^2}{2(G - \sigma)^2} \tag{4c}$$

$$= \left(\frac{1}{E} + \frac{\alpha_0^2}{G}\right)\sigma + \frac{1}{2}\frac{(\alpha_0^2 + k^2)\sigma^2}{G^2} + higher\ order \tag{4d}$$

Comparing (3) with (1) and (2) we see that the average shear strain of the two elements is equal to the shear strain that would be produced in a single element with zero initial tilt. Eq. (4d) implies that under pure compression the compressive strain is the same as that given by the 1-element model; the presence of a shear stress increases the compressive strain over that obtained from the 1-element model by the quantity involving k^2 in (4d).

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Before discussing the inelastic case we note the fact that γ_a and γ_b can also be obtained graphically, as shown in Figure 1. A simple geometrical argument can be given to show that the shear strains in the graph obey (1) and (2).

INELASTIC COMPOSITE

We make use of the same graphical construction when the shear stress-strain relation is nonlinear. Eqs. (4a) and (4b) remain valid for finding the compressive stress-strain relation. Fig. 3 illustrates schematically several different cases of proportional loading. Fig. 3a represents the case $k = 0$. In this case element a and element b are tilted equal amounts in opposite directions, so there is zero shear strain in the composite as a whole. When $0 < k < \alpha_0$ the two elements are tilted unequal amounts in opposite directions (Fig. 3b). As noted in Part I, the applied shear stress is the ordinate of the sloping line taken at the abscissa $-\alpha_0$. When $k > \alpha_0$ the elements are tilted unequal amounts in the same direction (Fig 3c). For $k >> \alpha_0$ they are tilted almost equally in the same direction (Fig. 3d), and the average strain is virtually the same as that of a single element with no initial tilt. This latter circumstance justifies the commonly accepted practice of ignoring fluctuations in fiber orientation when treating off-axis compression or tension for the case of large angles between fiber and load axis. Currently neither theory nor experiment is available for the case where the angle is small (the case treated in the present paper).

NUMERICAL EXAMPLE

To gain some feeling for stress interaction effects, we consider a numerical example. Fig. 4 shows the shear stress-strain curve for carbon-carbon discussed in Part I of this paper arbitrarily extended with the aid of a French curve to 4% shear strain. Such extension is needed to find the compressive stress-strain relations in the presence of shear all the way to failure, but since it was done in a rather arbitrary manner no claim is made for the accuracy of the resulting interaction curves.

Using the procedures outlined earlier in conjunction with Fig. 4, and with the assumption that $\alpha_0 = 0.030$, we obtain the results shown in Fig. 5. Four proportional loading cases are considered, viz. $k = 0$ (pure compression), $0.5\alpha_0, 2\alpha_0$, and ∞ (pure shear). The results indicate that the presence of even a very small shear stress will significantly reduce the axial stiffness and compressive strength of a composite, and a large axial compressive stress will greatly reduce the shear stiffness and strength.

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DISCUSSION

Most of the preceding discussion of the 2-element kink model has been concerned with proportional loading. Other loading paths can also be treated rather easily with the aid of the graphical construction outlined here. Note, however, that nonproportional loading may lead to unloading of one or other of the two kink elements. Thus to analyze nonproportional loading it may be necessary to know the pure shear behavior of the composite for unloading as well as for loading.

An alternative approach to the solution of an intrinsically nonlinear problem such as that treated here is to use an incremental linear theory. The justification for this is that virtually any curve can be approximated to arbitrary precision by a series of sufficiently short straight lines. Such an approach has been proposed by Jortner [2].

Let us analyze the problem under consideration from this point of view. The composite is assumed to be subject to combined stresses τ and σ , and to have its fibers tilted by a small angle α relative to the compression axis. We first add an increment of compressive stress $d\sigma$ holding the applied shear stress constant. The corresponding increment in the shear stress parallel to the fibers is given by

$$d\tau_1' = \alpha \, d\sigma + \sigma \, d\alpha_1 \tag{5}$$

where $d\alpha_1$ is the change in tilt caused by the increment in compressive stress in the presence of shear. We then add an increment of shear stress $d\tau$ while holding the compressive stress constant, which leads to a second increment in tilt angle $d\alpha_2$ and the shear stress parallel to the fibers $d\tau_2$. The result is

$$d\tau_2' = d\tau + \sigma \, d\alpha_2 \tag{6}$$

Since we are employing a linear theory, the increments are additive, i.e., $d\tau' = d\tau_1' + d\tau_2'$ and $d\alpha = d\alpha_1 + d\alpha_2$. Thus

$$d\tau' = d\tau + \sigma \, d\alpha + \alpha \, d\sigma \tag{7}$$

Integrating, we obtain

$$\tau' = \tau + \alpha \sigma \tag{8}$$

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which is the nonlinear finite equation that serves as the fundamental building block of the 1-element model in the present theory.

As noted earlier, when $k >> \alpha_0$ a 1-element model is a good approximation to the behavior of the composite as a whole. For pure or nearly pure compression it is inadequate because (depending on the the choice of α_0) it either fails to account for the nonlinear stress-strain relation and low failure stress in on-axis loading or it leads to spurious shear strains.

The 2-element model is free of these difficulties. It leads to the generally accepted results for axial compressive strength, and with at least some of the data on the compressive stress-strain relation. Its predictions in regard to compression-shear interaction are in qualitative agreement with the available experimental data. However since data on interaction are very limited and exhibit a large amount of scatter, a quantitative evaluation of the accuracy of the theory must await new and better experiments.

REFERENCES

1. S.B. Batdorf and Robert W.C. Ko, "Stress-Strain Behavior and Failure of Uniaxial Composites in Combined Compression and Shear-I", UCLA-ENG-85-25, July, 1985.

2. J. Jortner, "Effects of Weak Interfaces on Thermo-structural Behavior of Carbon Carbon Composites". ONR Annual Report, JR&E 8526, 1985.
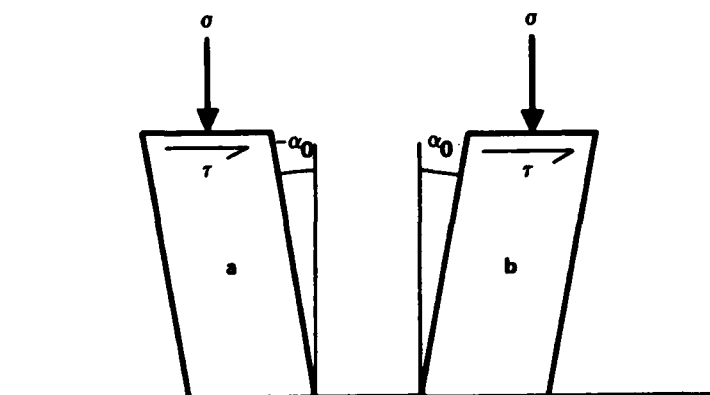
Fig. 1   A 2-element model.

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Fig. 2 Graphical method for finding shear strains in a elastic composite. Note $\overline{OA} = (\alpha_0 + k)\sigma$ and slope of $\overline{OC} = G - \sigma$. Thus, $\overline{AC} = \gamma_b$.
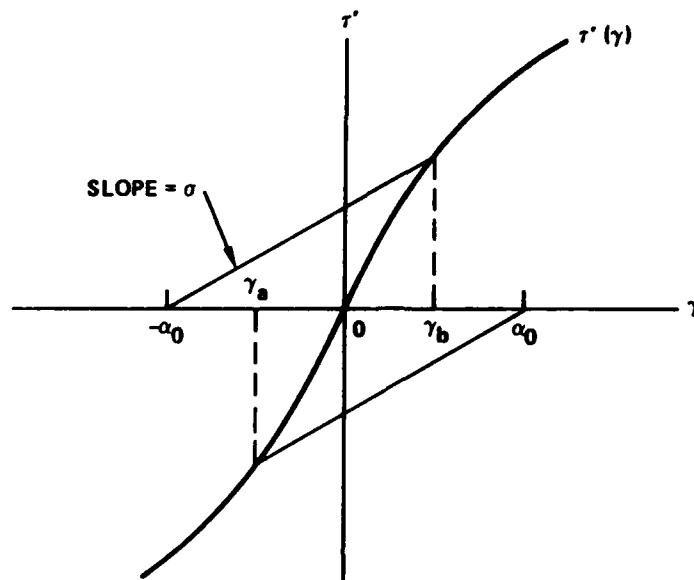
Fig. 3a Graphical method for proportional loading in a inelastic composite. In this pure compression case, $k = 0$, $\bar{\gamma} = 0$ for all σ.
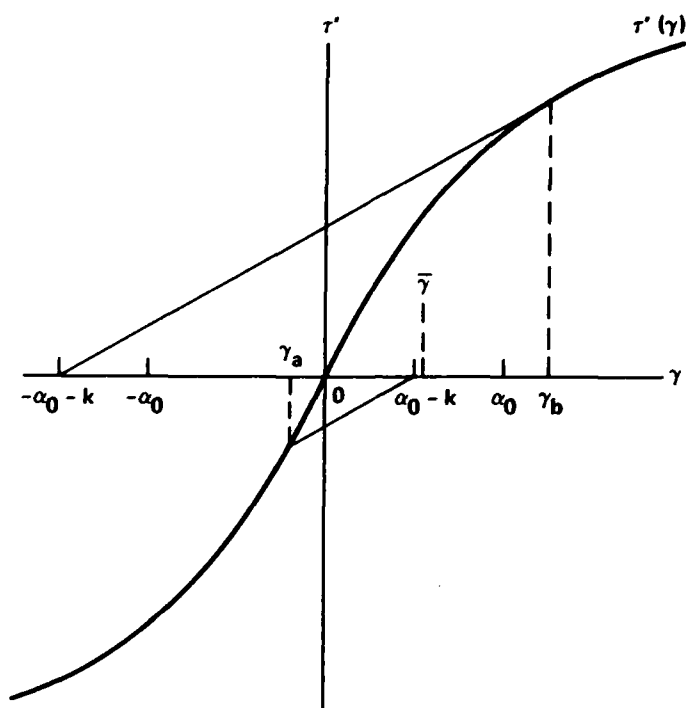
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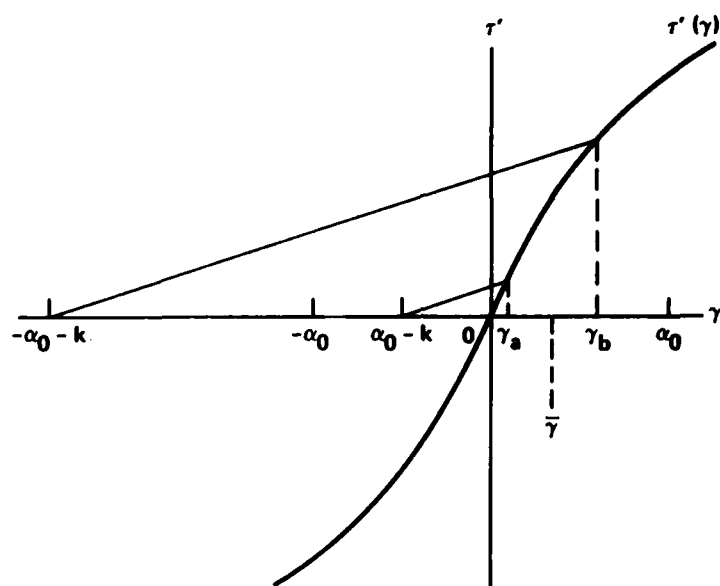Fig. 3b Graphical method for proportional loading, $0 < k < \alpha_0$.

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Fig. 3c Graphical method for proportional loading, $k > \alpha_0$.

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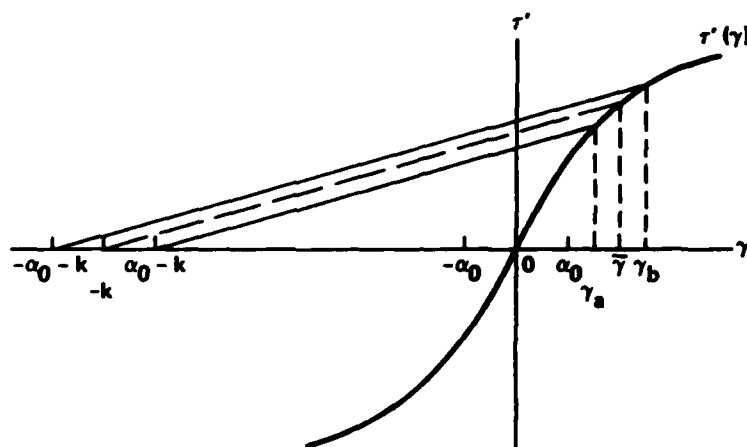
Fig. 3d Graphical method for proportional loading, k >> α_0.
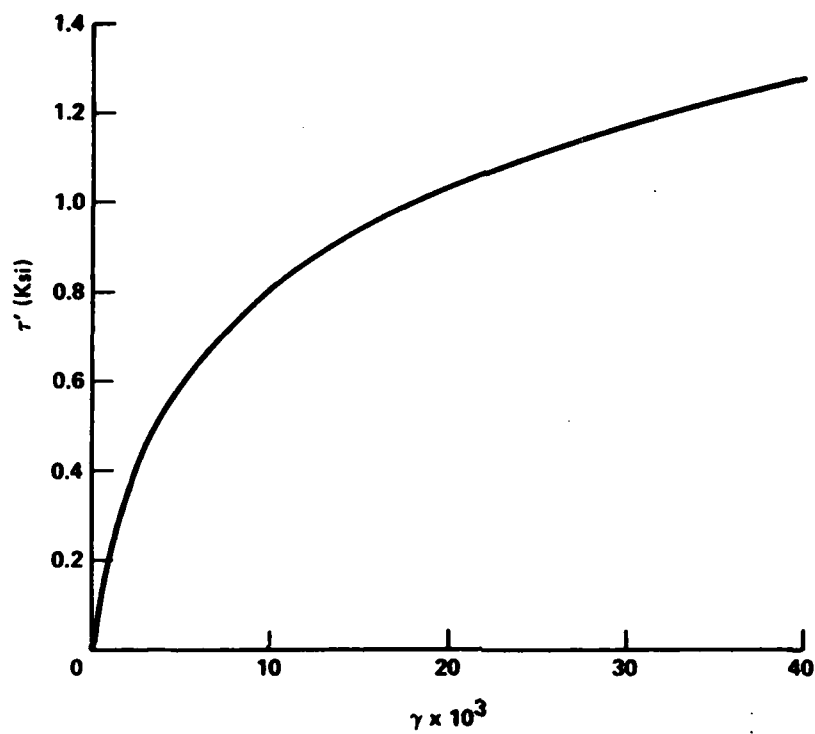Note $\overline{\gamma} = \gamma$ as if $\alpha_0 = 0$.

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Fig. 4   Shear stress-strain curve for carbon-carbon composites
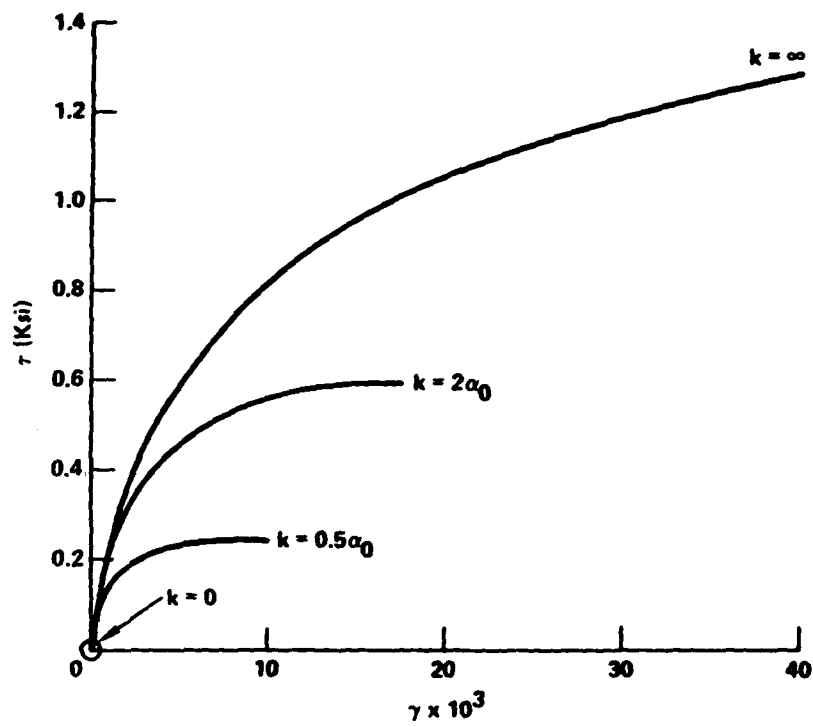with arbitrary extension after $\gamma = 0.02$.

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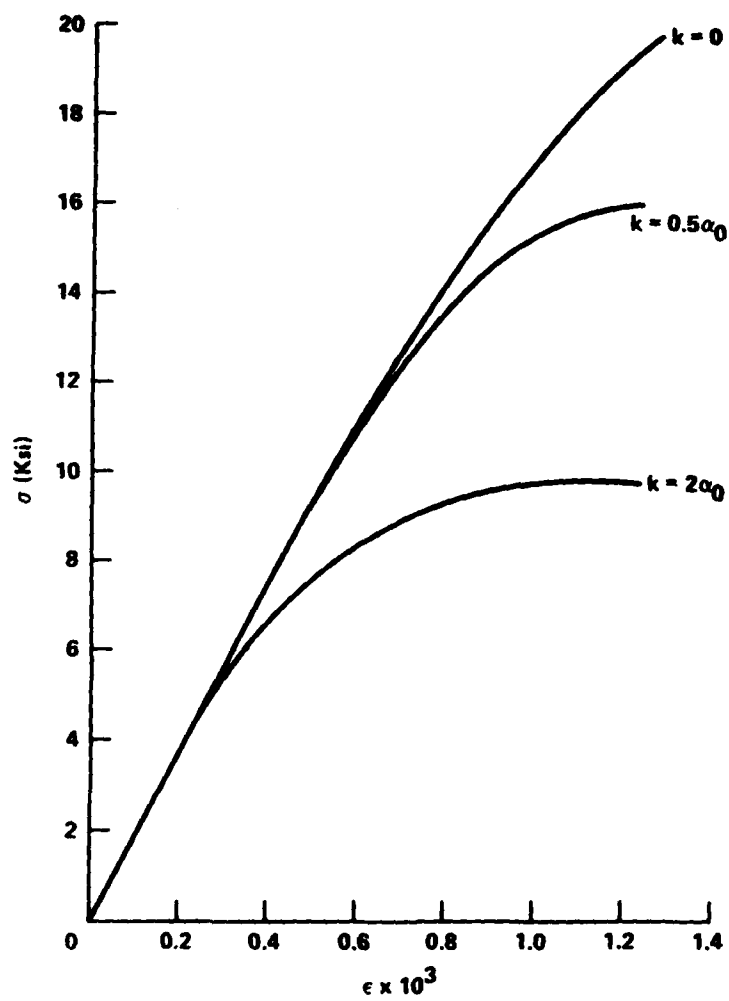Fig. 5a   Shear stress-strain relations under combined loads.

Fig. 5b Compressive stress-strain relations under combined loads.

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